The Binary Cascade

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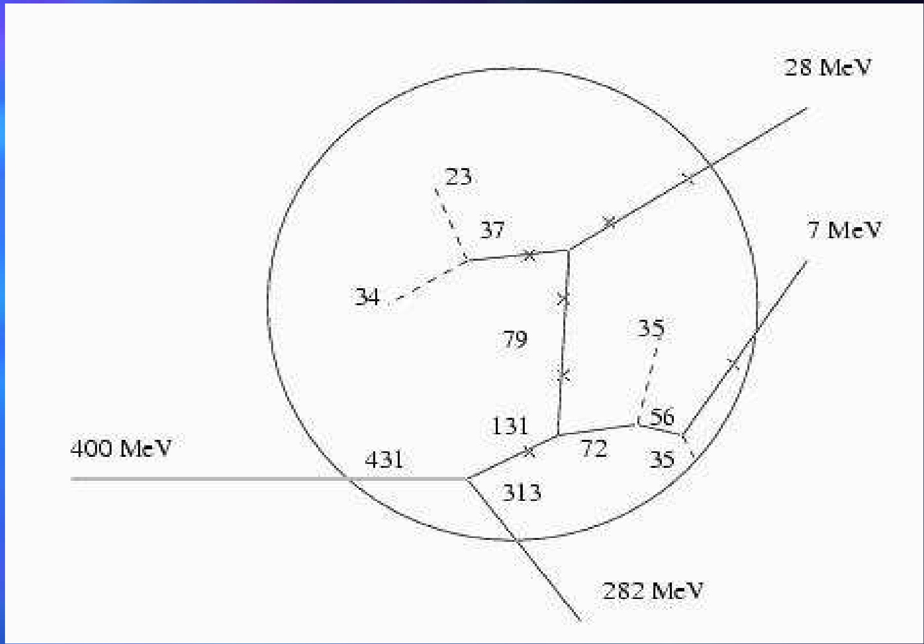
Intra-nuclear transport

- Initial notes:
  - In inelastic particle nucleus collisions, there is a phase of fast particle emission ($10^{-23} - 10^{-21} s$).
  - A Bolzmann equation needs to be solved to treat the physical process of the collision in detail.
  - The original intra-nuclear cascade model developed by Bertini solves the Bolzmann equation on average.
  - Much later other cascade-type models have been proposed to the same purpose.

Cascade type models



- Bertini and generalized cascade
- Quantum molecular dynamics (QMD)
- Binary cascade

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Some history of the intra-nuclear cascade

- First proposed by R. Serber:
  - Nuclear reactions at high energies, Phys.Rev.72,v11,p1114f(1947)
- Statistical calculations by M. Goldberger
  - The interaction of high energy neutrons and heavy nuclei, Phys.Rev.74,v10, p1269ff(1948).
- Monte Carlo computer code by Metropolis et al
  - Monte Carlo simulations of inter-nuclear cascades, I, II, Phys.Rev.110,v1, p185ff, p204ff, 1958.
- 1963 H.W. Bertini published standard methods to be used in many cascade implementations
  - Phys.Rev.131,p1801, 1963 (ORNL-3383)

Canonical components of an intra-nuclear cascade

- A description of the nucleus
  - A, Z, densities, etc..
- A description of the scattering of moving particles off nucleons in the nucleus
  - Cross-sections, angular distributions
- A description of the coherent interaction of moving particles with the nuclear field.
  - Hamiltonian, potential, refraction/reflection
- A description of the effects of the nuclear medium on scattering cross-sections.
  - Pauli's principle, effective masses and width, etc..

Binary cascading

- Some characteristics of binary cascading:
  - In binary cascading, like in QMD, each nucleon participant is described by

$$\phi(x, q_i, p_i, t) = (2/(L\pi))^{3/4} \exp(-2/L(x - q_i(t))^2 + i p_i(t) x)$$

  - And the total wave function is assumed to be the direct product of these (no anti-symmetrization).
  - The equations of motion for this wave-form are identical in structure to the classical Hamilton equations, and can be solved by numerical integration.
  - In QMD, the Hamiltonian is calculated from 2- and 3-body interactions of the particles in the system. In Binary cascade, the Nuclear Hamiltonian is calculated from optical potentials based on the target nucleus' property.

The nuclear model

- A 3-dimensional model of the nucleus is constructed
  - Explicitly positioning individual nucleons
  - In local density approximation
  - Using the Fermi gas model
- The sampling is done in a correlated manner
  - such the local phase-space densities stay within what is allowed by Pauli's exclusion principle, and
  - such that the sum of all nucleon momenta equals zero.

The nuclear density

- The nuclear density distributions used are the Saxon-Woods form for high A

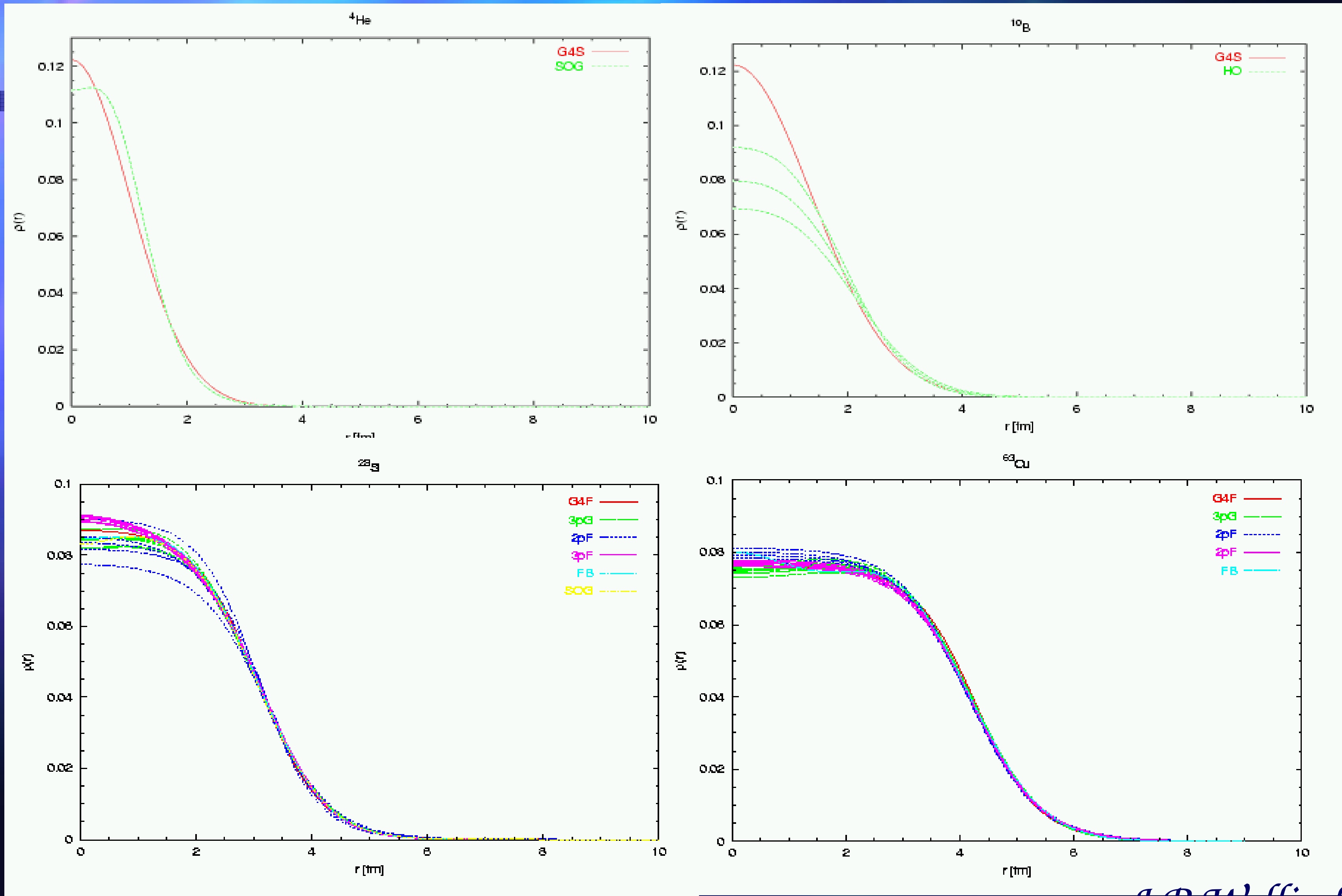
$$\rho(r_i) = \frac{\rho_0}{1+\exp[(r_i - R)/a]}, \quad \rho_0 = \frac{3}{4\pi R^3}\left(1+\frac{a^2\pi^2}{R^2}\right)$$

- Here $a = 0.545\, fm, \quad R = r_0 A^{1/3}, \quad r_0 = 1.16(1-1.16 A^{2/3})\, fm$
- And the harmonic oscillator form for light nuclei (A<17)

$$\rho(r_i) = (\pi R'^2)^{-3/2} \exp(-r_i^2 / R'^2)$$

- with $R' = \frac{2}{3} < r^2 > = 0.8133 A^{2/3} fm^2$

Nuclear densities: Ex. 4He, 10B, 28Si, and 63Cu



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The transport algorithm

1. The impact parameter is chosen randomly across the projected area of projectile and target nucleus.
2. The path of all cascade particles is projected as straight lines
    1. Based on geometrical interpretation of the free cross-sections.
3. The collision is simulated, and the final state is subject to Pauli's principle.
    1. If rejected, the procedure is repeated with the next collision in time.
4. If accepted, the system is transported in the nuclear potential, using a 4'th order Runge-Kutta integration method, and we go to step 2.

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The imaginary part of the G-matrix

- Acts as a scattering term
- It is described as 2-body, point-like collisions, resonance decay, and S-wave pion absorption
  - Collision assumption of black disk cross-section

Why binary cascade?

- The name binary cascade comes from the fact that only binary collisions (and decay) are considered, like

$$pp \rightarrow \Delta_{1232} N^*_{2190}, \text{ or } K^- p \rightarrow \Sigma^0{}_{1775}$$

- No further details on the mathematics, but the nucleon and delta resonances taken into consideration are these
  - Δ_{1232}, Δ_{1600}, Δ_{1620}, Δ_{1700}, Δ_{1900}, Δ_{1905}, Δ_{1910}, Δ_{1920}, Δ_{1930}, Δ_{1950}
  - N_{1400}, N_{1520}, N_{1535}, N_{1650}, N_{1675}, N_{1680}, N_{1700}, N_{1710}, N_{1720}, N_{1900}, N_{1990}, N_{2090}, N_{2190}, N_{2220}, N_{2250}

Channel cross-sections

- Many cross-sections in meson-nucleon scattering can be modeled as resonance formation in the S-channel.

$$\sigma_{res}\left(\sqrt{s}\right)=\sum_{FS}\frac{2J+1}{(2S_1+1)(2S_2+1)}\frac{\pi}{k^2}\frac{\Gamma_{IS}\Gamma_{FS}}{\left(\sqrt{s}-M_R\right)^2+\Gamma^2/4}$$

- Where the partial width for decay then will depend on the stochastic mass of the resonance

$$\Gamma_{R\to 12}(M)=(1+r)\frac{\Gamma_{R\to 12}(M_R)}{p(M_R)^{2l+1}}\frac{M_R}{M}\frac{p(M)^{2l+1}}{1+r\left[p(M)/p(M_R)\right]^{2l}},\quad r=0.2$$

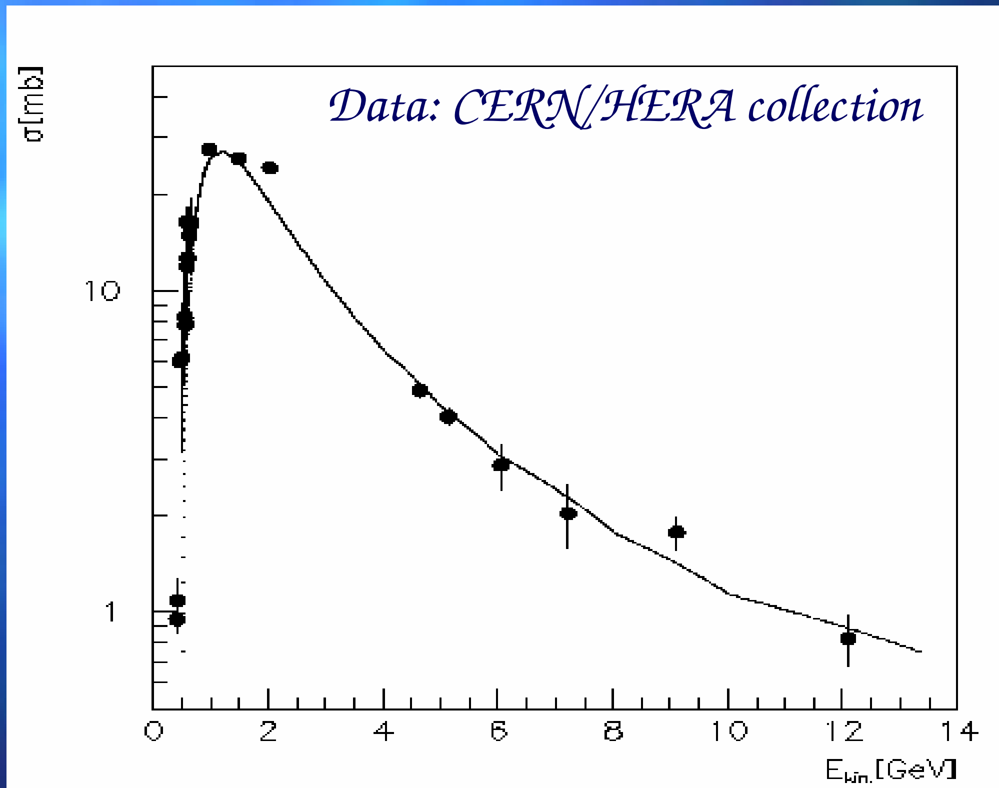
Resonance production in the t-channel

- The cross-section for t-channel resonance formation was parameterized from experimental data. The parameterization is motivated from the form of the Δ(1232) cross-section.

$$\sigma_{pp\to AB} = 2\alpha_{AB}\beta_{AB} \frac{\sqrt{s}-\sqrt{s_0}}{(\sqrt{s}-\sqrt{s_0})^2+\beta_{ab}{}^2}\left(\frac{\sqrt{s_0}+\beta_{AB}}{\sqrt{s}}\right)^{\gamma_{AB}}$$

- Includes single and double resonance excitation in nucleon nucleon scattering.

Predicting the Delta++ production cross-section in pp scattering by binary casacde



Non pp induced t-channel cross-sections

- Other cross-sections are derived from this using detailed balance.
  - Also isospin invariance is assumed

$$\sigma_{cd\to ab} = \sum_{J,M} \frac{\langle j_c m_c j_d m_d \| JM \rangle^2}{\langle j_a m_a j_b m_b \| JM \rangle^2} \frac{(2S_a+1)(2S_b+1)}{(2S_c+1)(2S_d+1)} \frac{\langle p_{ab}^2 \rangle}{\langle p_{cd}^2 \rangle} \sigma_{ab\to cd}$$

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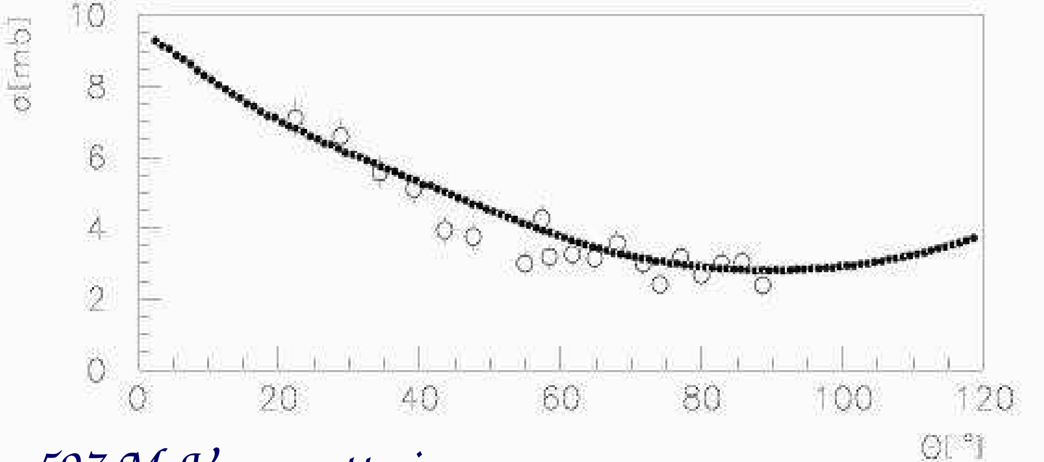
Nucleon nucleon elastic scattering final states

- Are taken from the phase-shift analysis of R. Arndt, (Mod.Phys.A 18,449(2003))



597 MeV pp scattering

Angular distributions for resonance re-scattering in the t-channel

- are approximated analytically from the collision term of the in-medium, relativistic Boltzmann-Uehling-Uhlenbeck equation, via scaling of the center of mass energy:

$$\sigma_{NN\rightarrow NN}(s,t) = \frac{1}{(2\pi)^2 s}\left(D(s,t) + E(s,t) + (inverted \quad t,u \right)$$

- with

$$D(s,t) = \frac{\left(g^{\sigma}_{NN}\right)^4 \left(t-4m^2\right)^2}{2\left(t-m^2_{\sigma}\right)^2} + \frac{\left(g^{\omega}_{NN}\right)^4 \left(2s^2+2st+t^2-8m^2s+8m^4\right)}{2\left(t-m^2_{\omega}\right)^2}$$

$$+\frac{24\left(g^{\pi}_{NN}\right)^4 m^4 t^2}{2\left(t-m^2_{\pi}\right)^2} + \frac{\left(g^{\sigma}_{NN} g^{\omega}_{NN}\right)^2 \left(2s+t-4m^2\right)m^2}{\left(t-m^2_{\sigma}\right)\left(t-m^2_{\omega}\right)}$$

- and

$$E(s,t)=\frac{\left(g_{NN}^{\sigma}\right)^4\left(t(t-s)+4m^2(s-t)\right)}{8\left(t-m_\sigma^2\right)\left(u-m_\sigma^2\right)}+\frac{\left(g_{NN}^{\omega}\right)^4\left(s-2m^2\right)\left(s-6m^2\right)}{2\left(t-m_\omega^2\right)\left(u-m_\omega^2\right)}+$$

$$\frac{6\left(g_{NN}^{\pi}\right)^4\left(4m^2-s-t\right)m^4t}{\left(t-m_\pi^2\right)\left(u-m_\pi^2\right)}+$$

$$\frac{3\left(g_{NN}^{\pi}g_{NN}^{\sigma}\right)^2m^2\left(4m^2-s-t\right)\left(4m^2-t\right)}{\left(t-m_\sigma^2\right)\left(u-m_\pi^2\right)}+\frac{3\left(g_{NN}^{\pi}g_{NN}^{\sigma}\right)^2m^2t(t+s)}{2\left(u-m_\sigma^2\right)\left(t-m_\pi^2\right)}+$$

$$\frac{\left(g_{NN}^{\omega}g_{NN}^{\sigma}\right)^2\left(t^2-4m^2s-10m^2t+24m^2\right)}{4\left(t-m_\sigma^2\right)\left(u-m_\omega^2\right)}+\frac{\left(g_{NN}^{\omega}g_{NN}^{\sigma}\right)^2\left((t+s)^2-2m^2s+2m^2t\right)}{4\left(u-m_\sigma^2\right)\left(t-m_\omega^2\right)}+$$

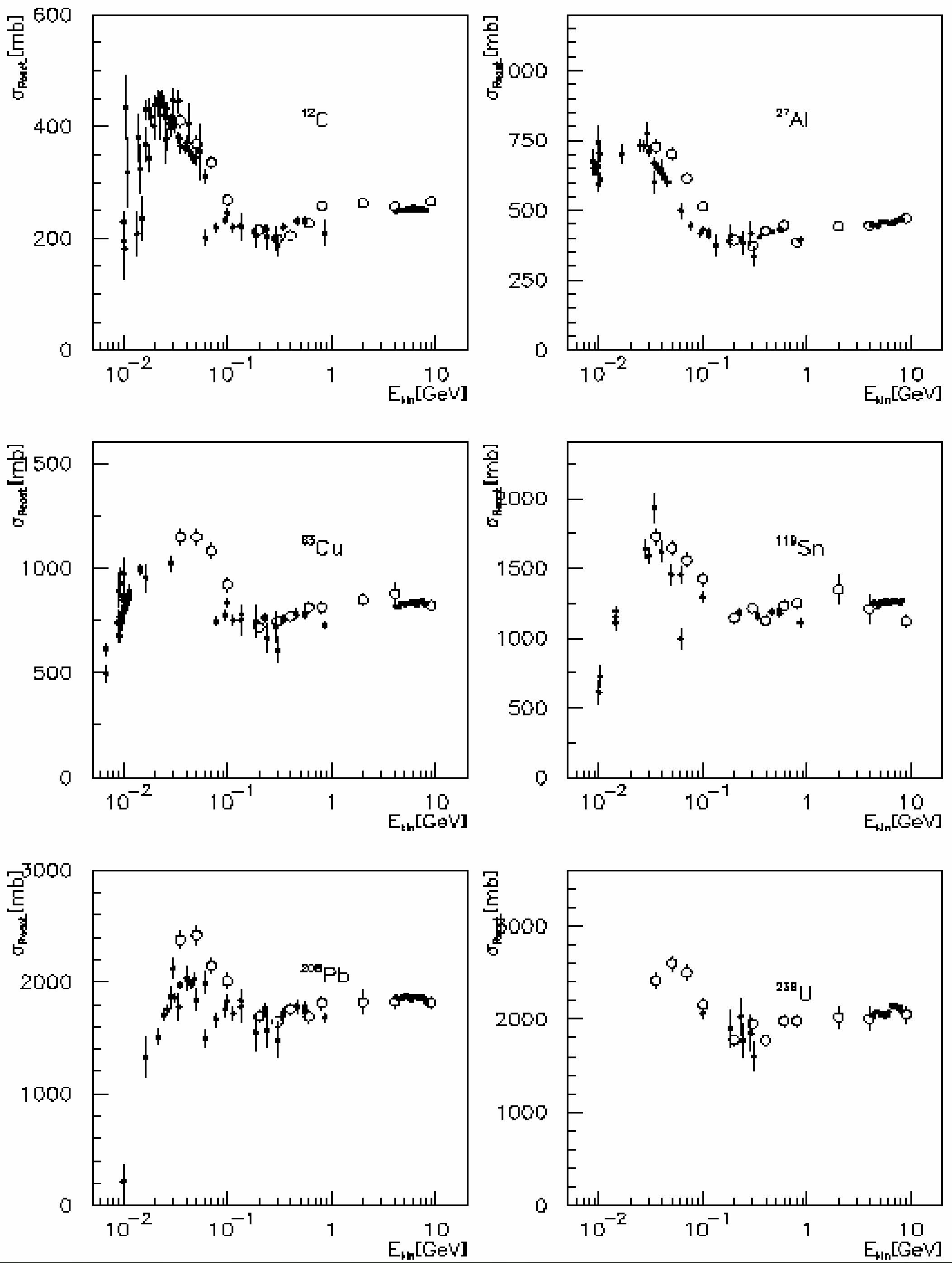
$$\frac{\left(g_{NN}^{\omega}g_{NN}^{\pi}\right)^2m^2\left(t+s-4m^2\right)\left(t+s-2m^2\right)}{4\left(t-m_\omega^2\right)\left(u-m_\pi^2\right)}+\frac{\left(g_{NN}^{\omega}g_{NN}^{\pi}\right)^2m^2\left(t^2-2tm^2\right)}{4\left(t-m_\pi^2\right)\left(u-m_\omega^2\right)}$$

Binary cascade prediction

Sample the impact parameter over a very large area.

Make the ratio of 'hits' to number of sampling attempts.

Multiply with the area sampled.



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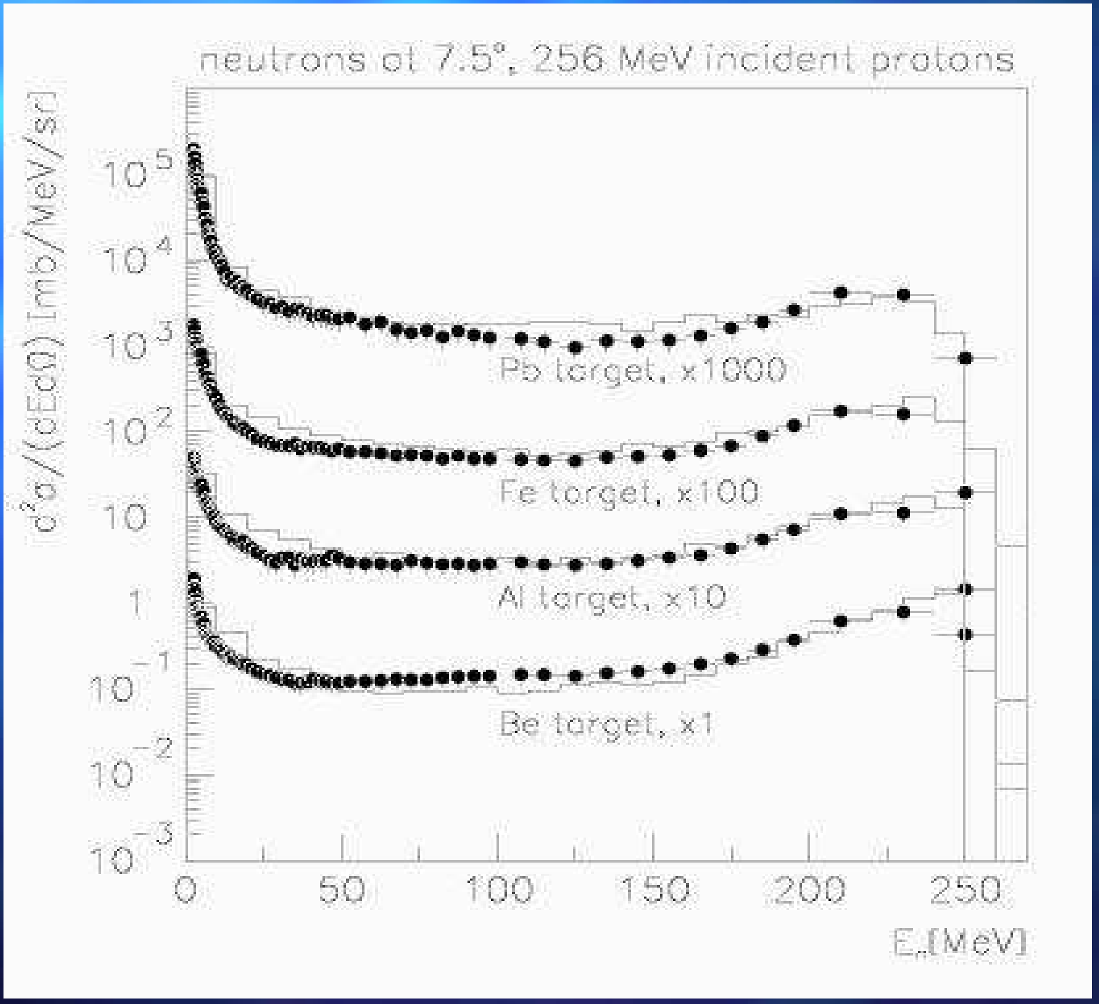
Forward scattering in proton scattering (256 MeV)



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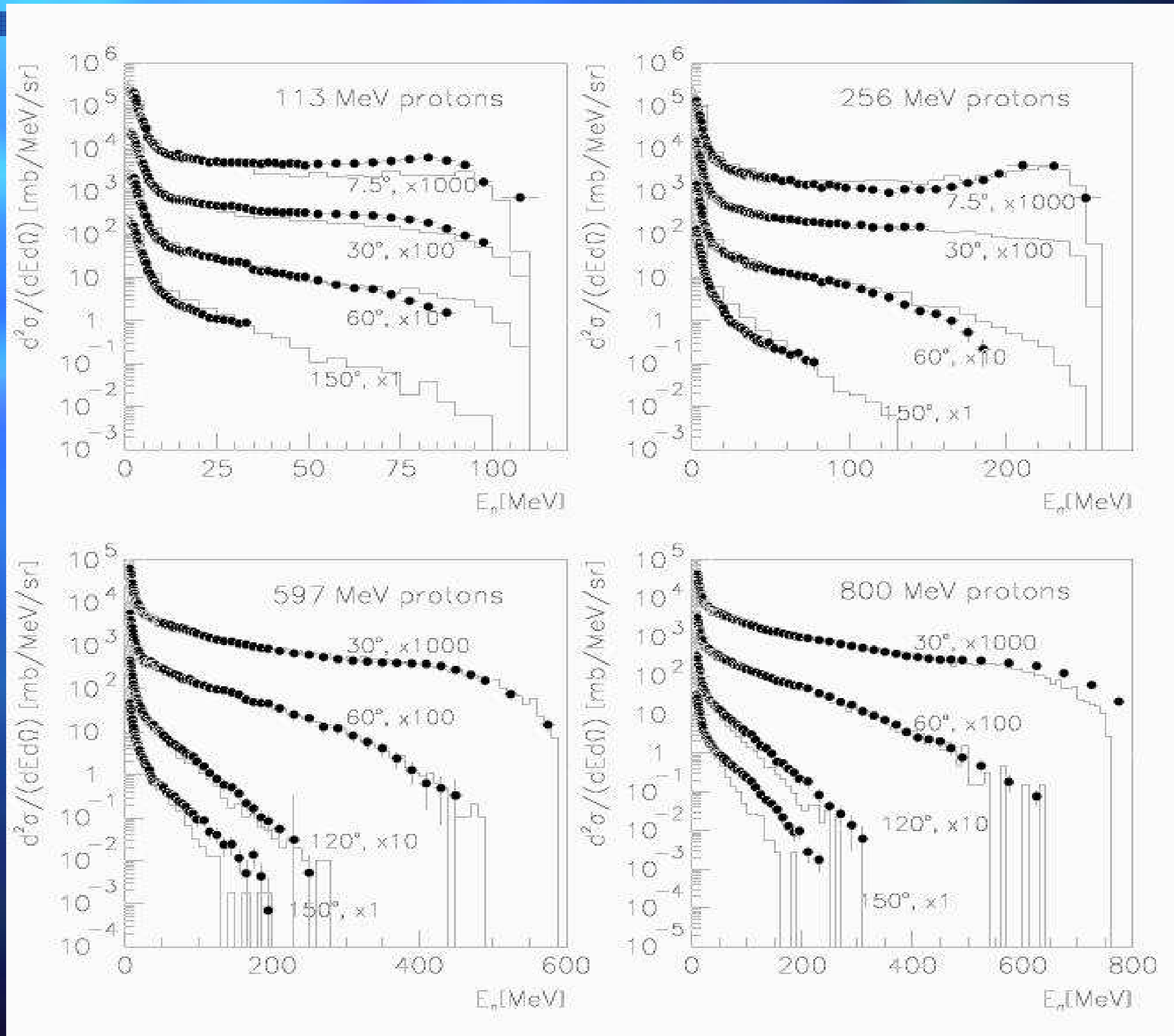
Scattering off lead at various angles and energies



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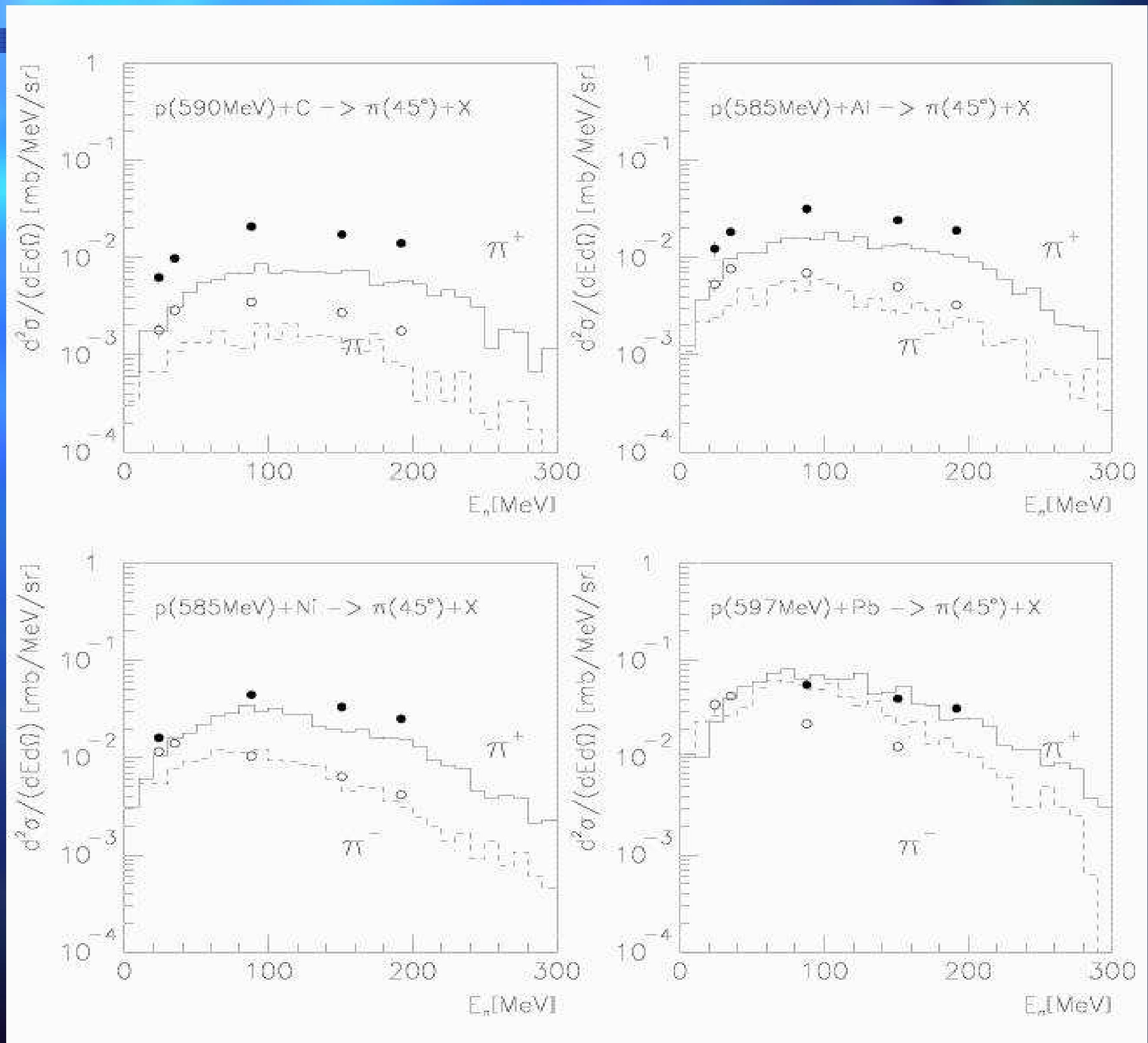
Pion production at ~600MeV on various targets



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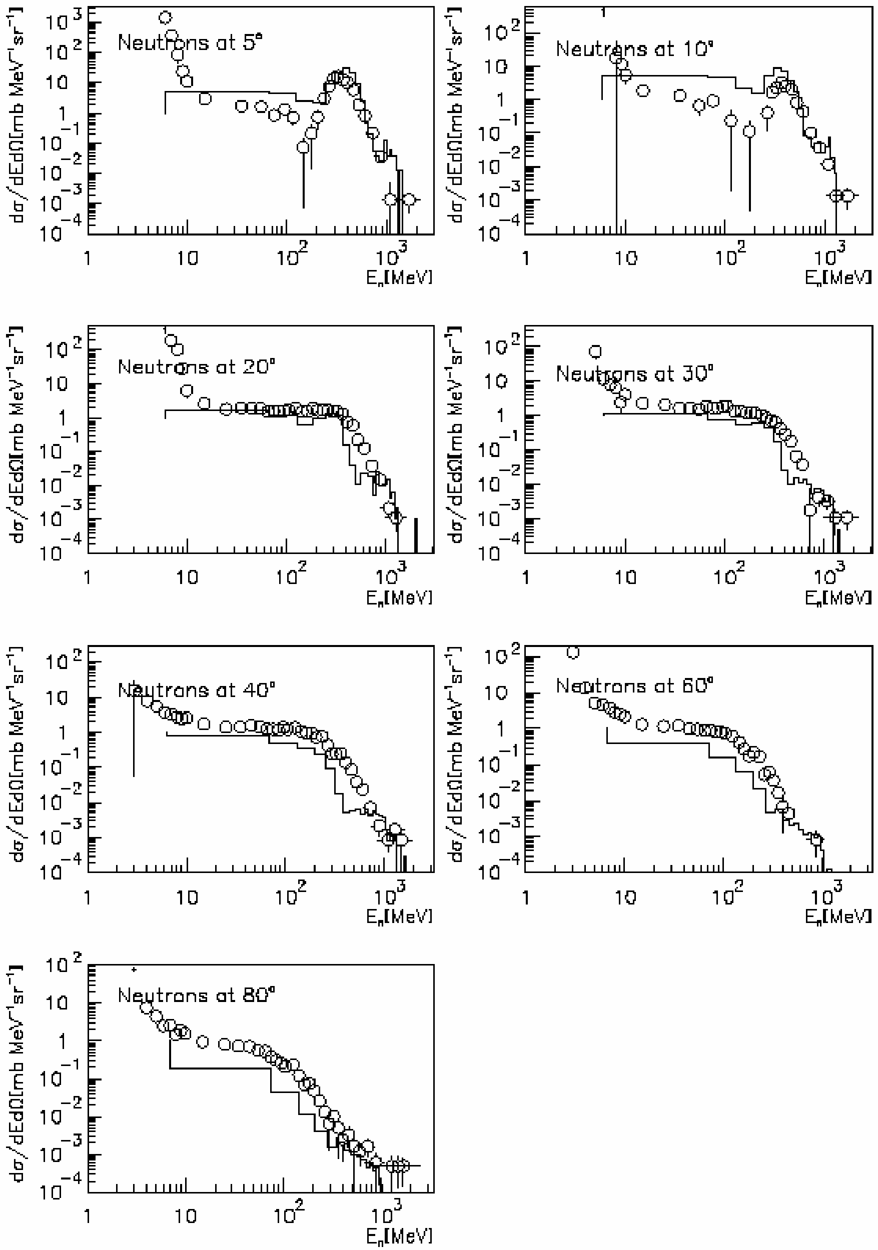
Neutrons from 400 MeV/A C-C reactions



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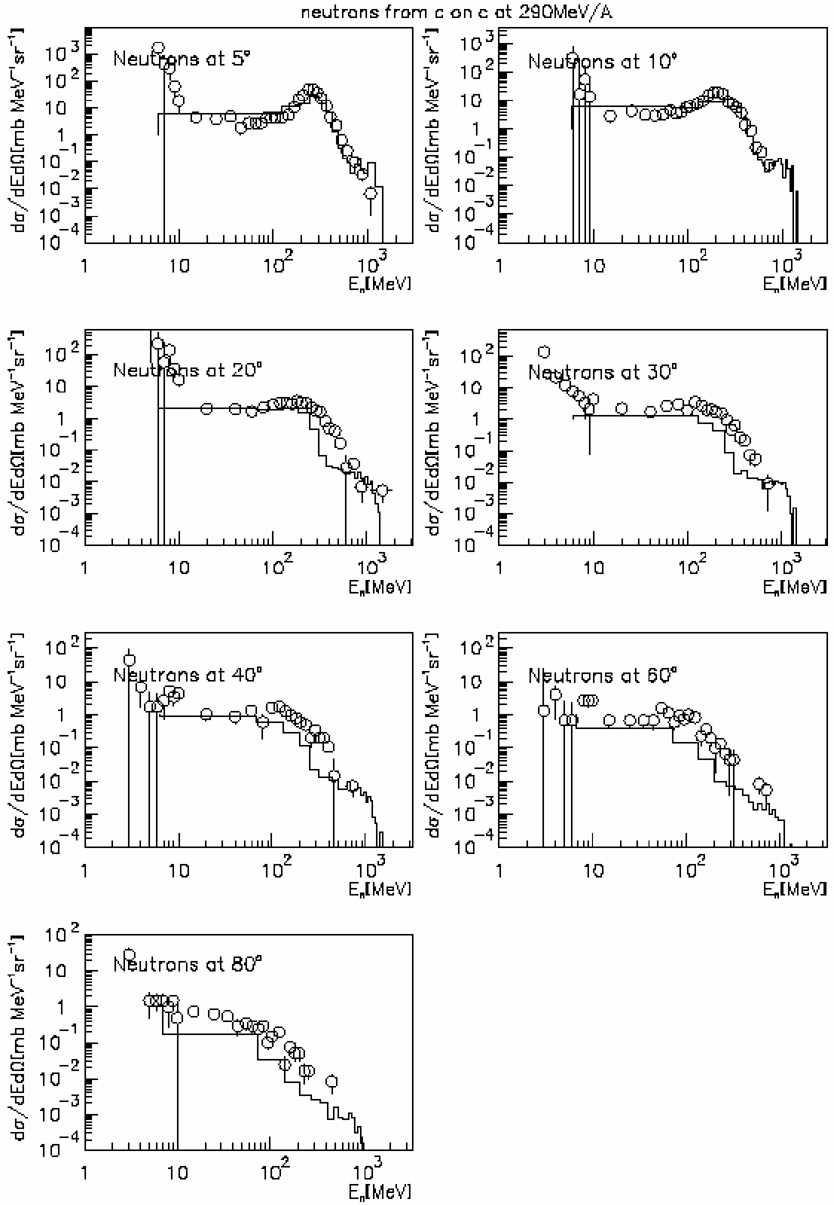

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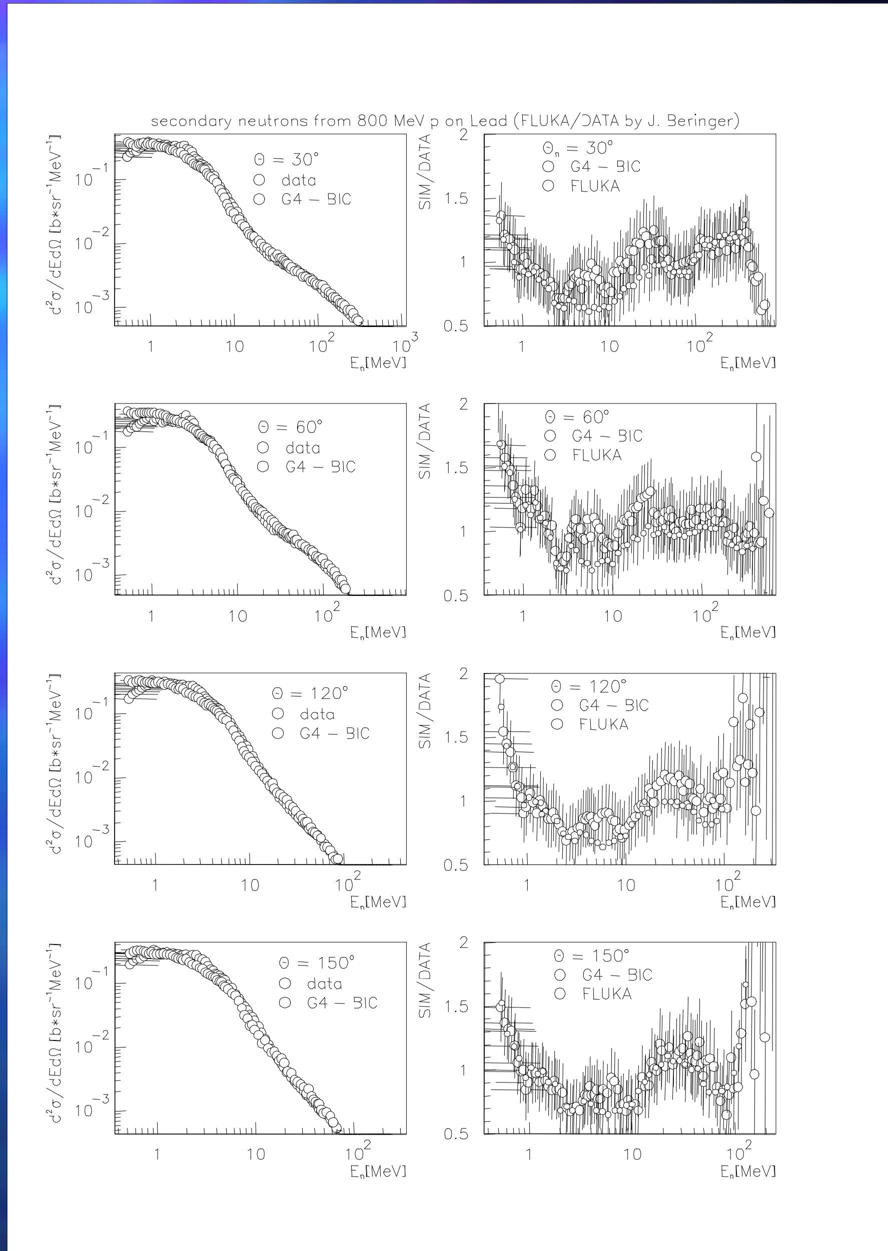
The latest, greatest



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Summary

- We have proposed a new approach for cascade simulations - Binary Cascade.
- The approach has been developed to describe particle production in nucleon and light ion induced reactions.
- We are adding gamma and R-hadron induced reactions.
- Most of all, the realization of the model is an excellent framework for trying out new theoretical ideas.